一人時間差相関
～ BHの直接検出を目指して ～

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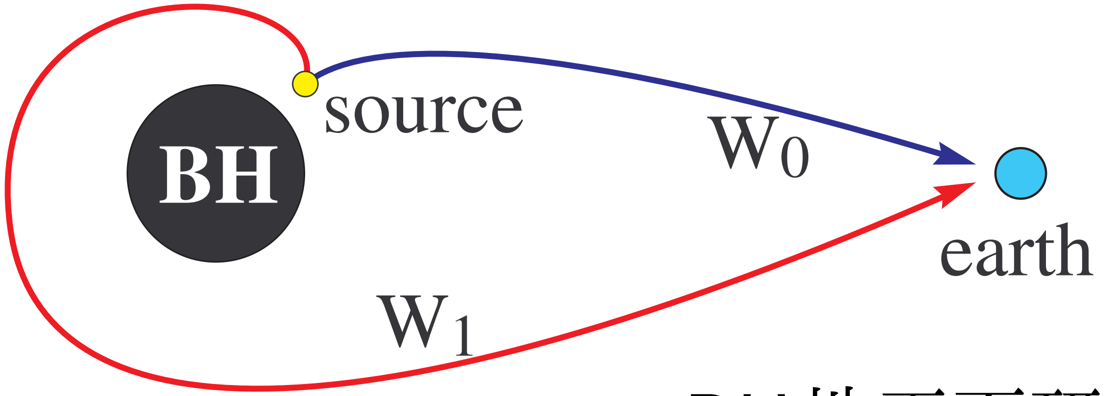


BH地平面研究会 at 山口, 2014.10.4-5
Subaru seminar at Hawaii, 2014.5.23

Plan of Talk

1. Introduction : Basic idea for direct BH detection
2. Proposal : A principle of direct BH detection method for one telescope
3. Under calculation : Short review of the aim of my current calculation
4. Summary

1. Introduction : basic idea

1.1 From candidate to itself

- Best observational knowledge of BH at present
  → BH candidates by Newtonian gravity

$\Updownarrow$ Large Gap in Physics !!

- BH is **a general relativistic (GR) object**
  → The only way to find "BH itself" is
  a direct detection of GR effect caused by BH.

What is it? How can we do it?

1.2 BH detection in GR context

- Theoretical (mathematical) fact in GR

Uniqueness (or No Hair) Theorem

BH is uniquely specified by 3 parameters (under physically suitable conditions in math. cal.):

M : mass
J : spin angular momentum
Q : electric charge

(No other parameter (hair) is assigned to BH.)

- BH detection in GR context is as follows:

  Qualitative meaning is (∵ BH is a GR object)
  to recognize the existence of BH
  by detecting GR effect.

$\Downarrow$ ∵ Uniqueness Thm

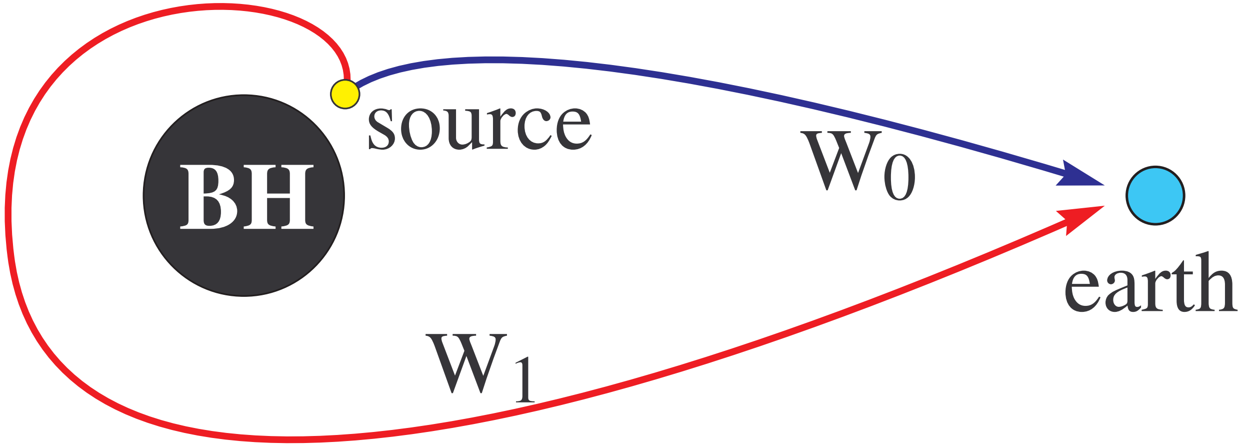
Quantitative meaning of BH detection

To measure the parameters M and J by detecting GR effect.

Note: $Q = 0$ is expected in real situations.

1.3 GR effect of BH as our target

- Target : **Strong Gravitational Lensing effect**
- An ideal situation we want to observe:
  - ◇ Clear environment around BH except the source
  - ◇ Burst-like and spherical emission seen from the source



W_0 : "0th ray" (direct ray)

W_1 : "1st ray" (secondary ray)

$\Downarrow$ our basic idea is simple!

- Basic idea of a direct BH detection:

From two observational quantities

$$\left\{\begin{array}{lll}\Delta t_{\mathrm{obs}} & : & \text{Time delay} \\ \mathcal{E}_{\mathrm{obs}} = \dfrac{E_1}{E_0} & : & \text{Amp. ratio}\end{array}\right. \text{ of } \mathrm{W}_0 \text{ and } \mathrm{W}_1 \text{ ,}$$

Obtaine two BH parameters M and J .

- Method to observe $(\Delta t_{\mathrm{obs}}\, , \, \mathcal{E}_{\mathrm{obs}})$ should be **realizable by one telescope.**

2. Principle of direct BH obs.

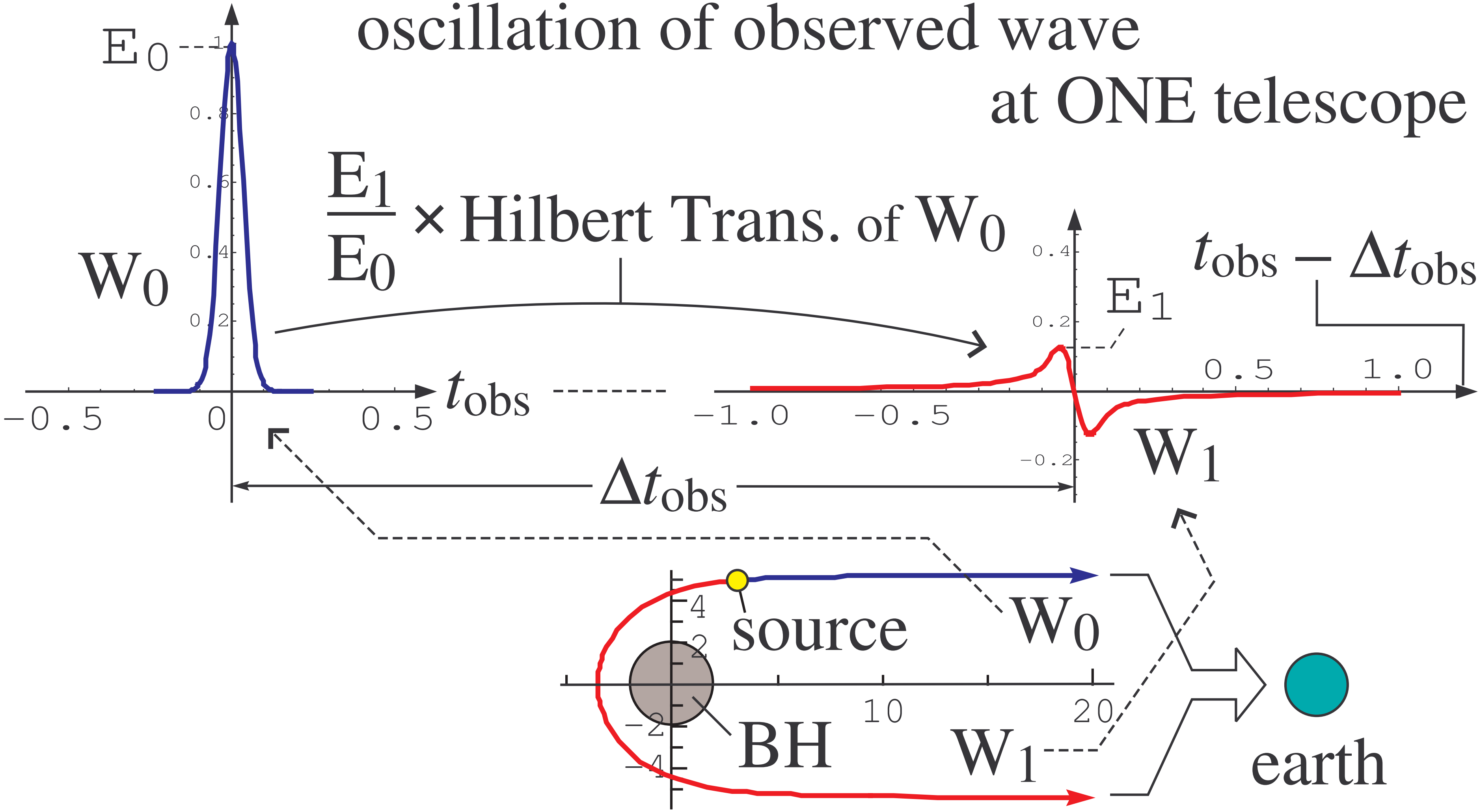
- Making use of time series data of one telescope

2.1 Ex. of time series data on one ideal tele.

- Suppose:
  The emission of source is Gaussian in time.
- Suppose:
  The telescope detects the time-variation of electric field (or its intensity) at all wave length.

$\Downarrow$ Then $\cdots$

- Gaussian emission → **waveform changes !**



Ref: Zenginoglu &Galley PRD86(2012)064030 , YouTube

- **Gouy Phase Shift** : General phenomenon of wave

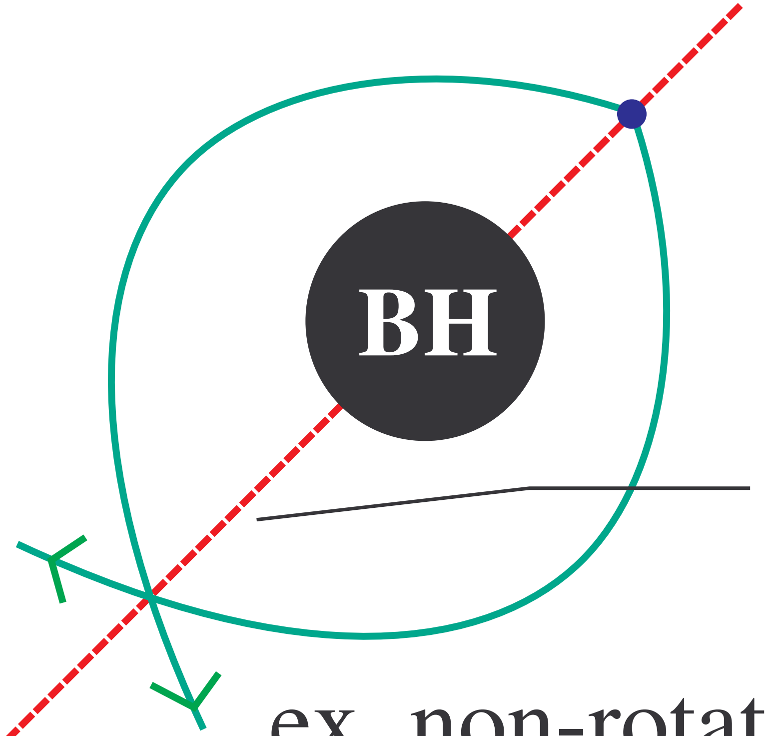
  The phase of oscillation shifts unexpectedly,

  when the wave passes a caustic.

  (when some rays cross there.)

  ∗ Caustic

  = Crossing points of rays



ex. non-rotating BH

Math.: Wave opt. approx. breaks down.

Higher order approx. reveals this effect.

$\rightarrow$ When some rays of wave cross at a caustic,

$$\begin{cases} \text{Positive freq. Fourier component:} \\ \qquad \text{Phase shifts by } -\dfrac{\pi}{2} \text{ [rad]} \\ \text{Negative freq. Fourier component:} \\ \qquad \text{Phase shifts by } +\dfrac{\pi}{2} \text{ [rad]} \end{cases}$$

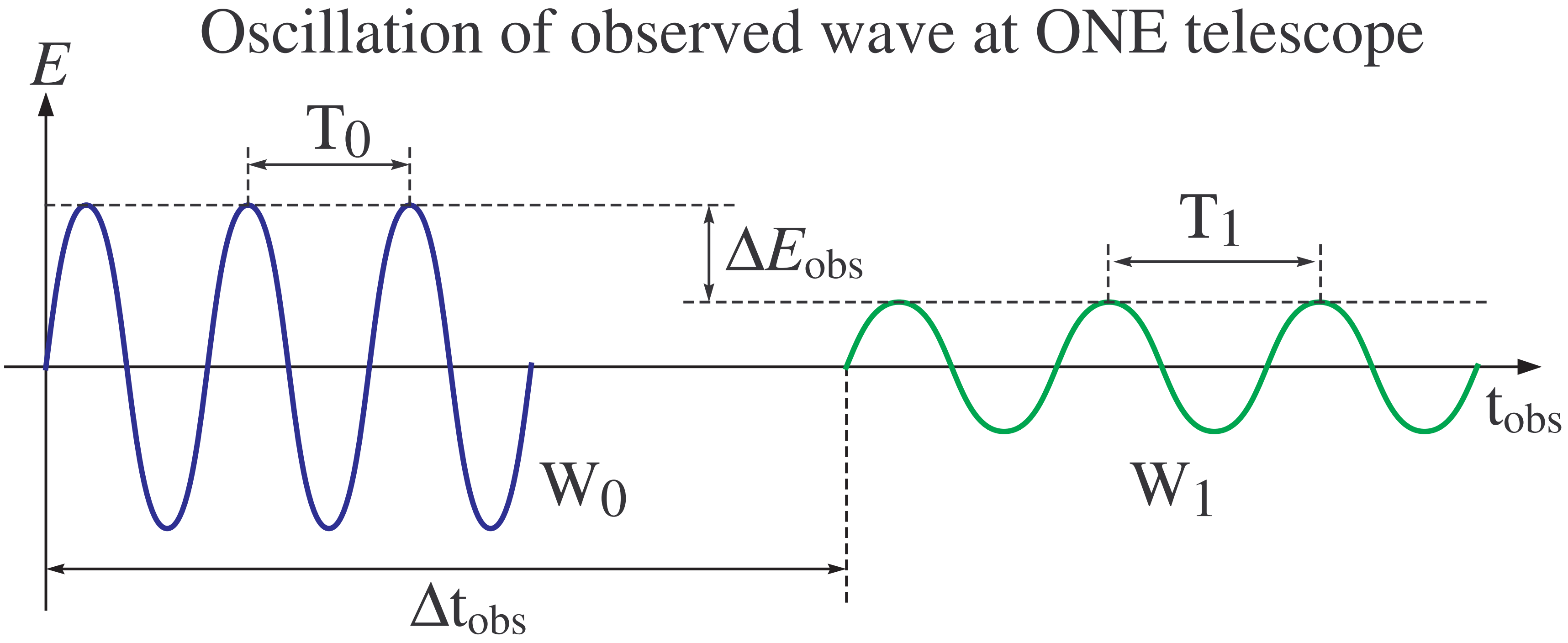
Note: Spectrum remains unchanged.

Math.: This is expressed by Hilbert trans.

$$\text{a wave } f(t) \xrightarrow{\textbf{Hilbert}} H[f](t) \;\propto\; \mathrm{Re} \int_{-\infty}^{\infty} \mathrm{d}z \, \frac{f(z)}{t-z}$$

2.2 Ex. of time series data with line emission

- Suppose: An exact line emission by the source

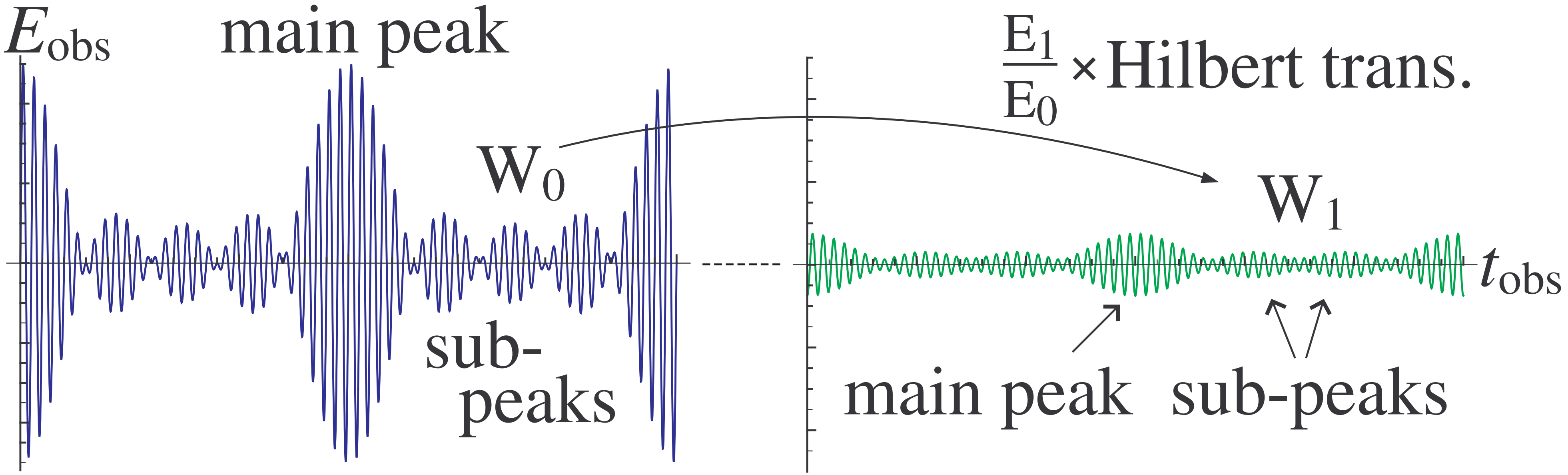


$$\rightarrow \begin{cases} T_0 \neq T_1 \text{ due to kinematic Doppler effect.} \\ \text{Gouy phase shift } H[\sin](\omega t) = \cos(\omega t) \end{cases}$$

2.3 Ex. of an observation with a band width

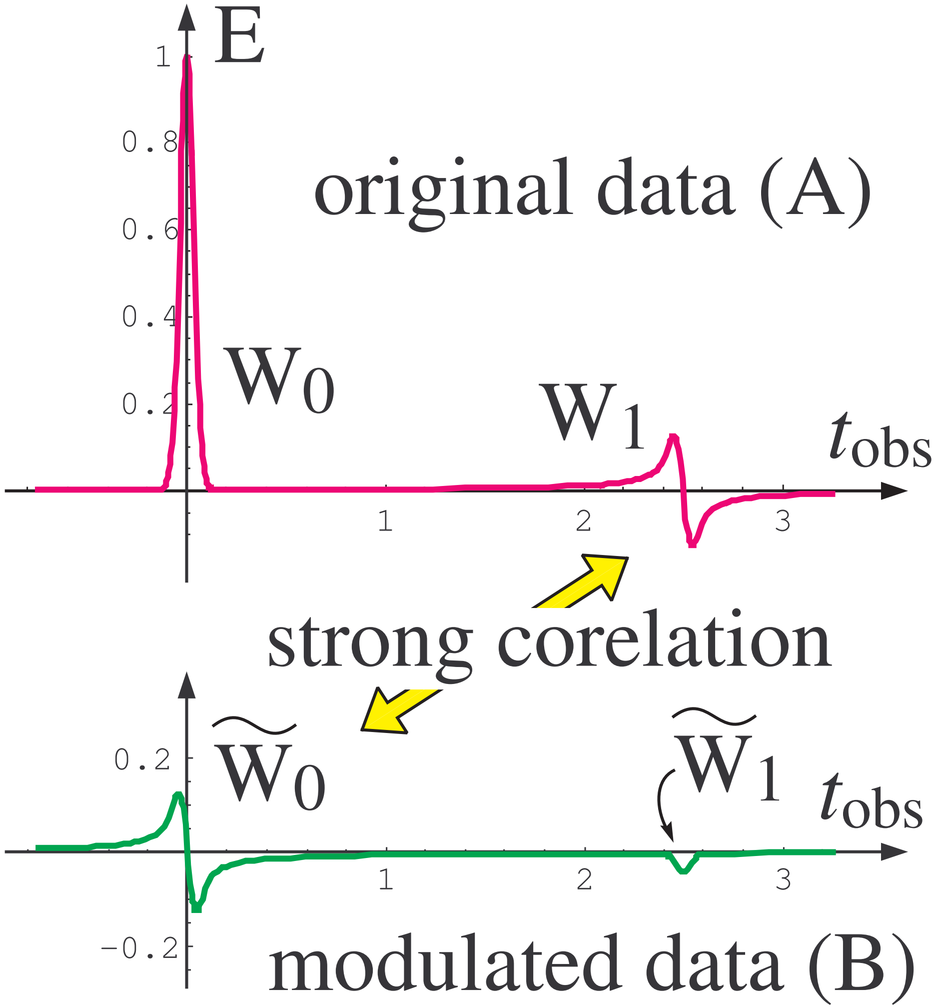
- Waveform in time series data is a **Beat**

$$\text{Ex. } W_0 = \sum_{n=0,\pm1,\pm2} \sin[\,(\omega + n\,\delta\omega)\,t\,]\;, \quad \begin{cases} \omega = 2\pi \\ \delta\omega = \dfrac{2\pi}{30} \end{cases}$$



→ Waveform-change may be apparent in beating.

2.4 Time Delay Self-correlation (TDS) method



Step1: Data copy (A, B)

Step2: Modulation of B

- ◇ Hilbert trans. of B
- ◇ B × Constant ($\mathcal{E}_{\mathrm{obs}}$)
- ◇ Doppler correction on B

Step 3: Correlation search
between A and B

→ W_0 and W_1 are found,
and Δt_{obs} , $\mathcal{E}_{\mathrm{obs}}$, T_1/T_0 are obtained.

- An actual case:
  - ◇ W$_0$ and W$_1$ are covered with background noise.
  - ◇ The background noise is a random oscillation.

→ Background noise vanishes in Correlation Integral.

$$\int \mathrm{d}\tau \, N(t)N(t+\tau) = 0 \ \text{ for a random noise } N(t)$$

→ Non-random signals, W$_0$ and W$_1$, are to be obtained by Step 3.

3. Under calculation (theory)

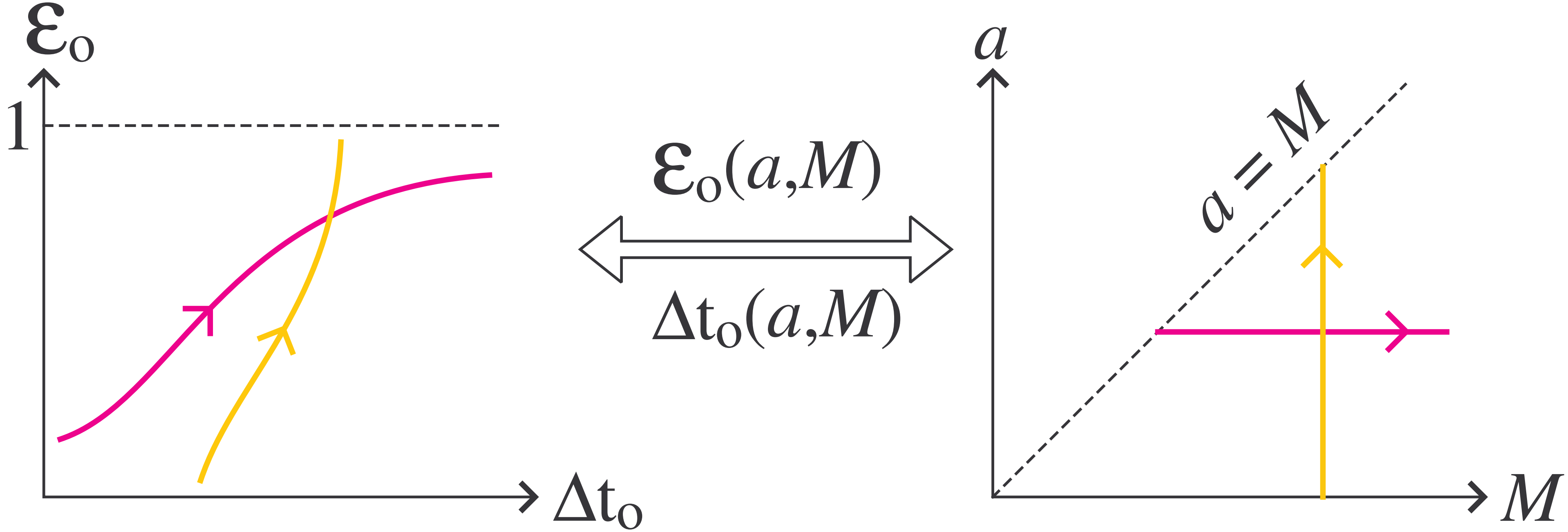
3.1 Correspondence $(\Delta_{\rm obs}, \mathcal{E}_{\rm obs}) \leftrightarrow (M, J)$

- Suppose the values:

$$\begin{cases} \text{source position : } (\, t_{\rm s}\, , \, r_{\rm s}\, , \, \theta_{\rm s}\, , \, \varphi_{\rm s}\,) \text{ at emission} \\ \text{source velocity : } (\, u^t_{\rm s}\, , \, u^r_{\rm s}\, , \, u^\theta_{\rm s}\, , \, u^\varphi_{\rm s}\,) \text{ at emission} \\ \text{emission spectrum : } I_{\rm s}(\nu_{\rm s}) \text{ seen from the source} \\ \text{inclination angle : } \theta_{\rm obs} \\ \text{observation frequency : } \nu_{\rm obs} \end{cases}$$

$\Downarrow$

under calculation with General Relativity

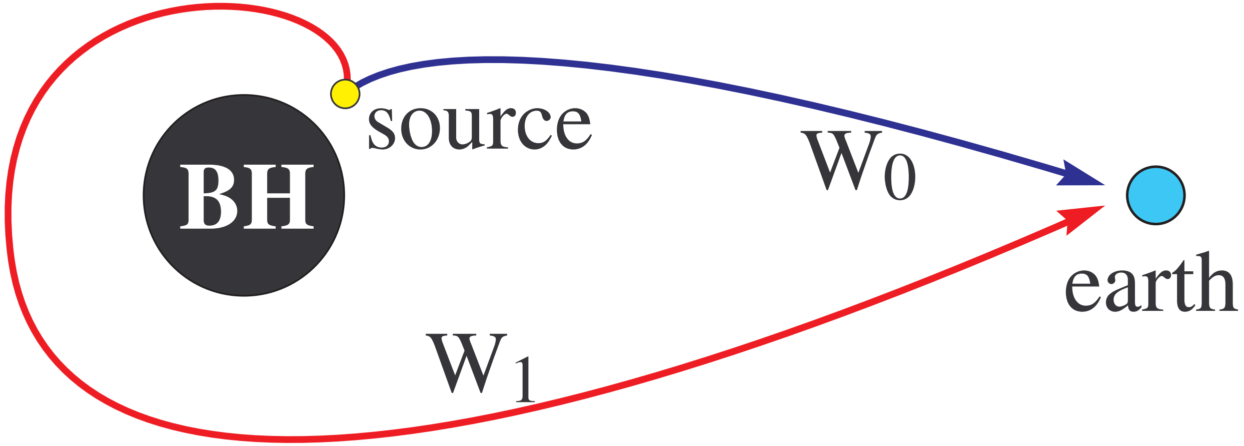


by definition : $\mathcal{E}_o < 1$, $0 < \Delta t_o$, $a < M$

This diagram enables us to read (M, J)
from observational data $(\Delta t_{\mathrm{obs}}, \mathcal{E}_{\mathrm{obs}})$

- いま計算で試行錯誤（四苦八苦）していること：

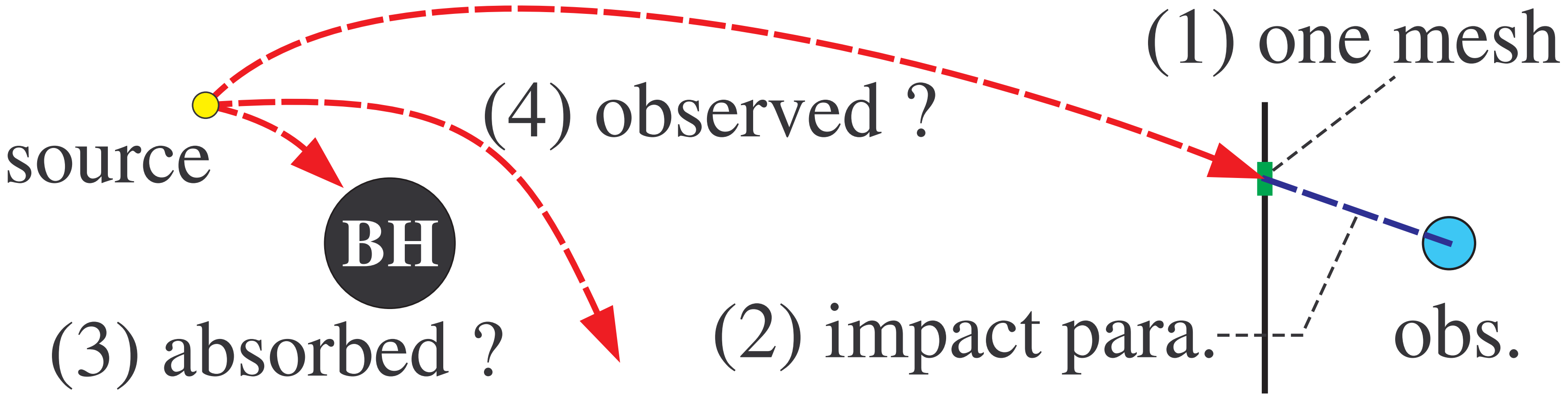
如何に $\mathbf{W}_0$ ，$\mathbf{W}_1$ を数値的に探すか？



◇ 常微分方程式の境界値問題：

$$\left\{\begin{array}{l}\text{光線がBHに突っ込むと計算がしんどい（解決）}\\ \text{光線の初期条件の探査がしんどい（未解決）}\end{array}\right.$$

◇ いまのプログラムの計算方法



(1) スクリーン上のメッシュを選ぶ。

(2) そのメッシュを通る光線の衝突因子が決まり，
光線の初速度も決まる。（ $\because$ $(k_r)^2 + V_{\rm eff} = 0$ ）

(3) 光線がBHに吸収されるか診断。（有効ポテンシャル）

(4) 吸収されないなら，測地線方程式を解き，観測者に
届くかどうか判断。（メッシュ幅エラーの範囲内）

→ 斉田のPC（CPU 2.6GHz , メモリ 16GB）を1日走らせても,
観測者に届く光線が見つからない …

○ メッシュをもっと細かくして,
計算時間を長くすれば見つかる？

○ プログラム改善の余地あり !?（最近バグとり終了）

愚痴：欲しいのはW_0とW_1の2メッシュなので,
ほとんどのメッシュが無駄メッシュだな〜 …
Shooting問題だと仕方ないか〜 …

→ 愚痴をこぼしたら,
別の計算方法に気付いた（**3**日前）

◇ 別の計算方法（検討中）: Shooting はしない

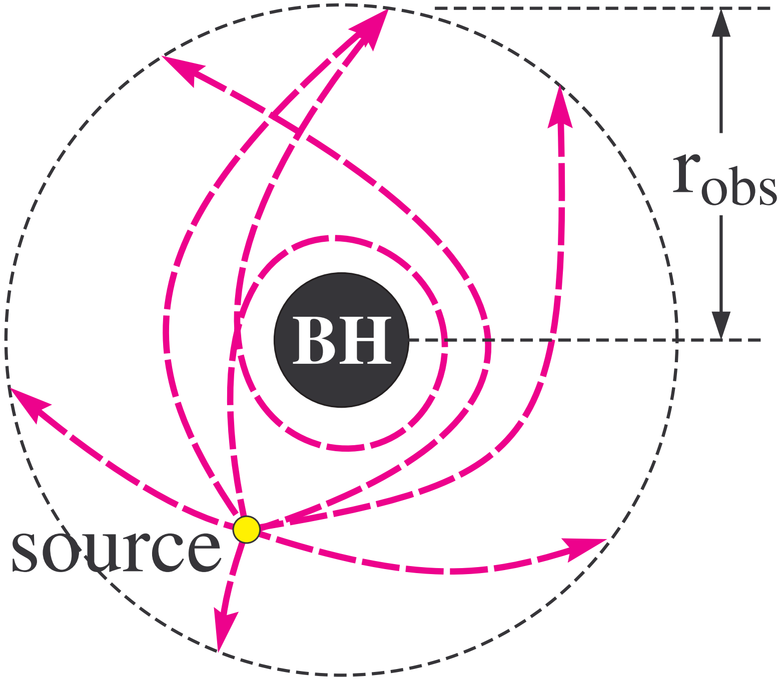
(1) 光源の位置を固定して，光線の初速度（の向き）を選ぶ。

(2) BH に吸収されるか診断。

(3) 吸収されないなら，r_{obs} まで測地線方程式を解く。

(4) 光線の巻き付き数と初期条件が，半径 r_{obs} の球面上の各点で記録できる。

(5) 球面上での観測者の位置は自由に選ぶ。



→ この方法だと …

球面上にメッシュを切り，それぞれのメッシュに
2本（以上）の光線が届くように

『初期条件を細かくずらした計算』

を繰り返す。

→ 計算時間はかかっても，

Shooting より無駄が少ないはず。

このプログラムに書き直そうかと思案中

この方が良さそうですよね???

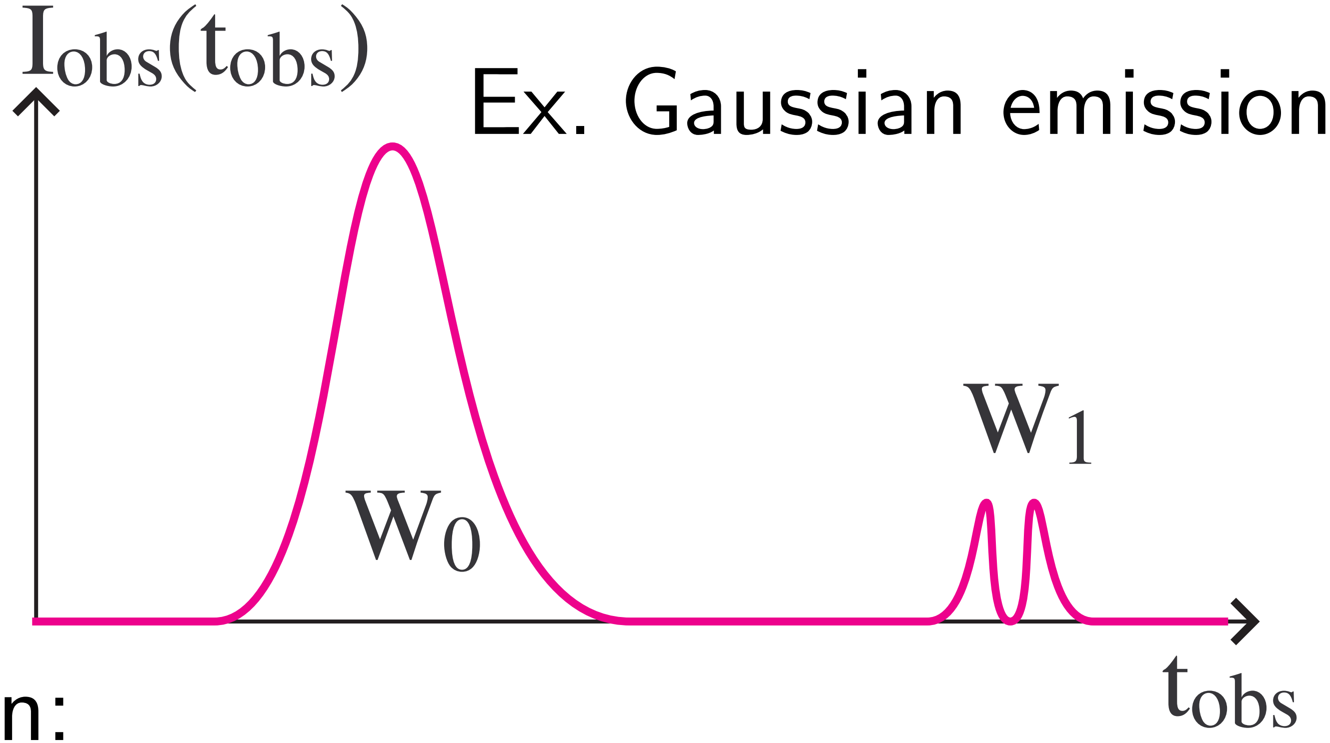
今ごろ気付くとは … 遅い …

3.2 TDS with Light Curve ?

$I_{\mathrm{obs}} \propto E^2 \rightarrow$

(E : Amplitude)

$\Downarrow$



Ex. Gaussian emission

Under consideration:

Mathematical transformation

between W_0's curve and W_1's curve.

$\rightarrow$ With this, the "step 2" is extended to light curve.

4. Summary

- Direct BH detection is to measure M and J via a direct observation of GR effect.
- For Strong Gravitational Lensing by BH, TDS method may realize the direct BH detection $\underline{\text{by one telescope.}}$
- Correspondence diagram $(M, J) \leftrightarrow (\Delta t_{\rm obs}, \mathcal{E}_{\rm obs})$ is under construction with GR.
- Extension of TDS to light curve, a construction of light curve trans. is also under consideration.