Black Hole Shadow について

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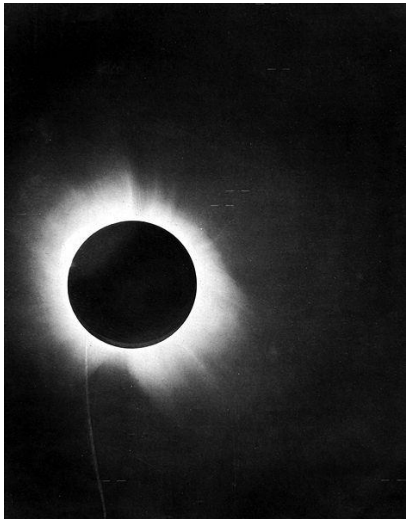
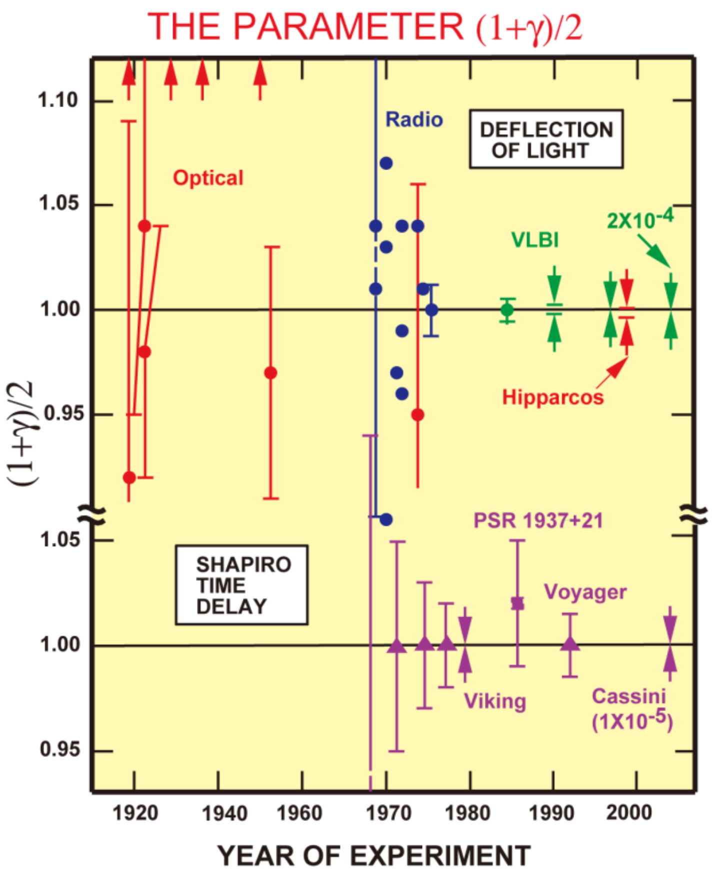
Kenta Hioki, & KM, Phys.Rev. D80, 024042, '09

General Relativity A. Einstein (1915)

First confirmed by A. Eddington (1919)

Light Bending at the Solar Eclipse





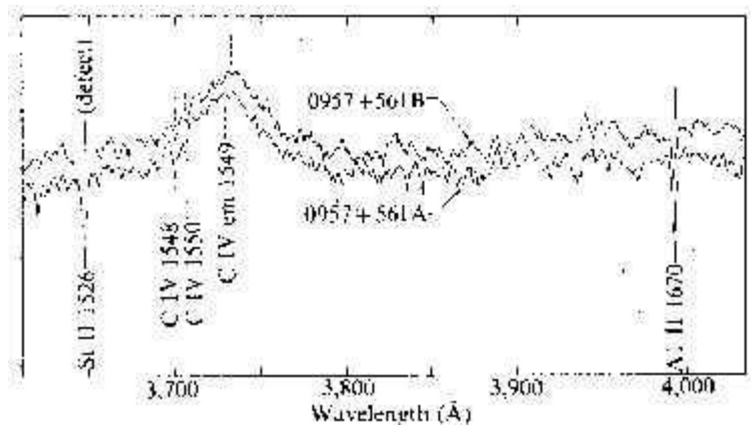
After ’60 (radio astronomy),
There are many observations.

C. Will,
Living Review, Relativity, 9, (2006), 3

Light Bending (Test of GR) ⇨ Gravitational Lensing (Astrophysics)

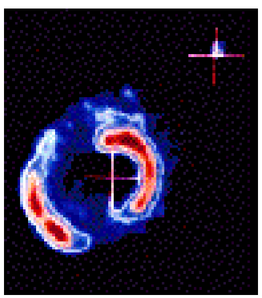
0957 + 561 A, B: twin quasars or gravitational lens?

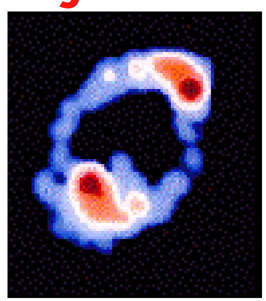
Walsh, Carswell, Weymann ('79)

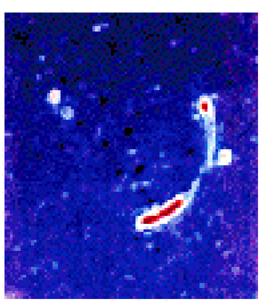


Many gravitational lens objects

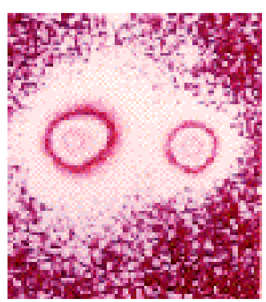
Einstein ring





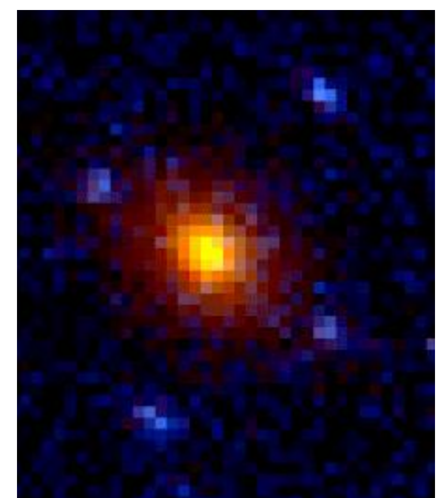


arc



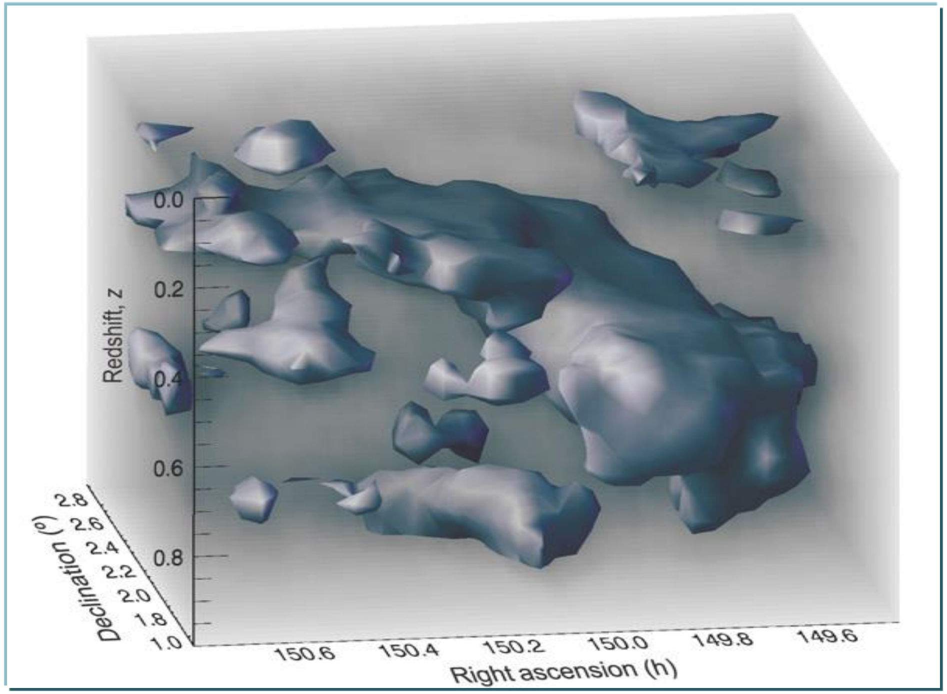
twin

ring



cross

Dark matter distribution



R. Massey et al, Nature 445, 286 ('07)

Perturbation of CMB anisotropies by gravitational lensing (Planck)

Mass Distribution back to the last-scattering

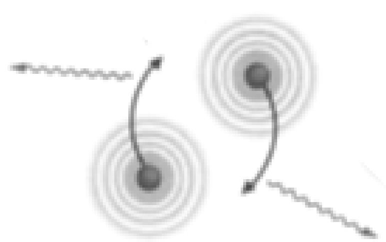
⇨ Cosmology

Black Hole

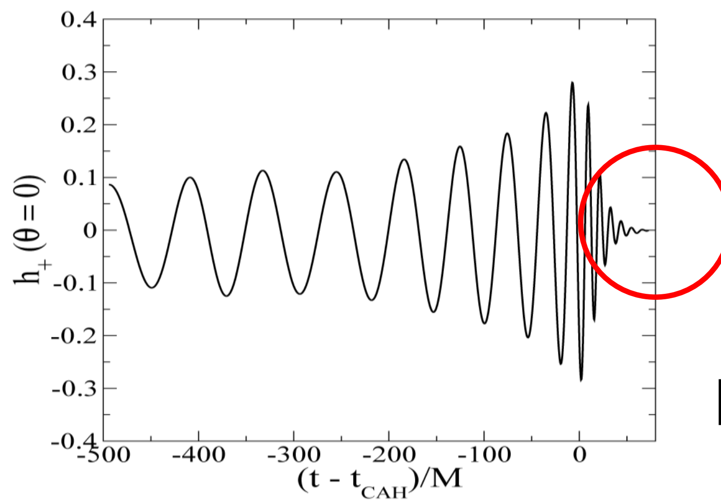
How to observe it ?

Direct Observation

Gravitational waves : Quasi-normal modes of a *black hole*



binary system



Pretorius ('07)

⇒ Formation of a black hole

Properties of a black hole (a spin)

Gravitational lensing : Black hole shadow

strong gravitational field near a black hole

⇨ strong gravitational lensing

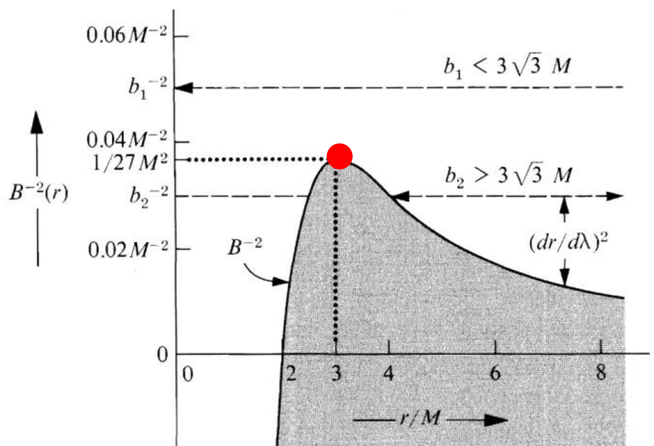
unstable circular photon orbit

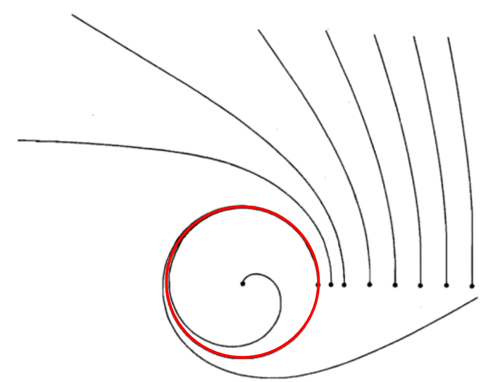
$$\left(\frac{dr}{d\lambda}\right)^2 + B^{-2}(r) = b^{-2};$$

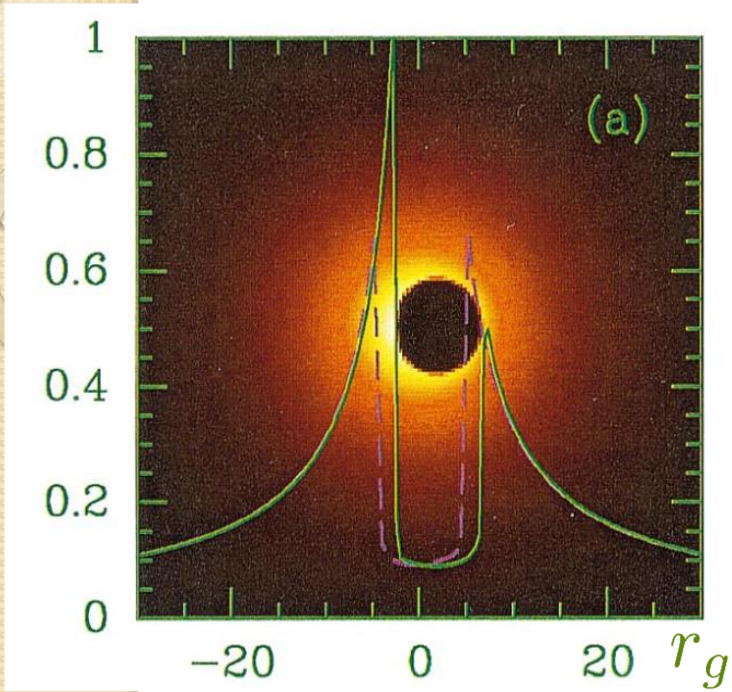
Schwarzschild BH

$$B^{-2}(r) = r^{-2}(1 - 2M/r);$$

$$b = \text{(impact parameter)}.$$

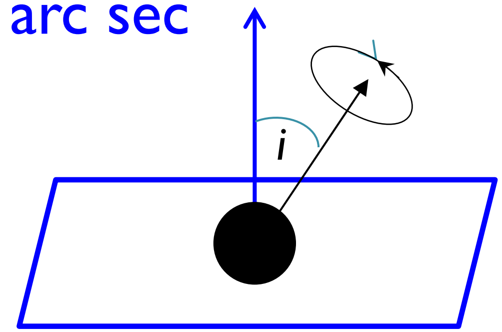






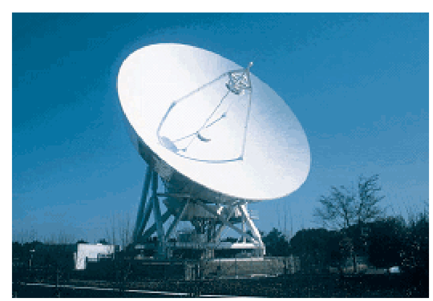
an optically thin emission region surrounding a black hole with **a=0.98M** in Sgr A* (Inclination i=45°)

horizon scale ~ 3 μ arc sec



Huang et al ('00)

Resolution can be achieved using VLBI



Kerr geometry in Boyer-Lindquist coordinates

$$ds^2 = -\left(1 - \frac{2Mr}{\rho^2}\right) dt^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 - \frac{4Mra\sin^2\theta}{\rho^2} dt d\phi + \frac{A\sin^2\theta}{\rho^2} d\phi^2$$

$$\Delta := r^2 - 2Mr + a^2 \qquad \rho^2 := r^2 + a^2\cos^2\theta \qquad A := (r^2 + a^2)^2 - \Delta a^2 \sin^2\theta$$

M : gravitational mass $\qquad J = Ma$: angular momentum

$a \leq M$ Kerr Black Hole

$a > M$ naked singularity

null geodesics

$$\rho^2 \frac{dr}{d\lambda} = \pm\sqrt{\mathcal{R}}$$

$$\rho^2 \frac{d\theta}{d\lambda} = \pm\sqrt{\Theta}$$

$$\rho^2 \frac{d\phi}{d\lambda} = \frac{1}{\Delta}[2Mar + \xi\csc^2\theta(\rho^2 - 2Mr)]$$

$$\rho^2 \frac{dt}{d\lambda} = \frac{1}{\Delta}(A - 2Mra\xi)$$

$$\mathcal{R} := \left(r^2 + a^2 - a\xi\right)^2 - \Delta\mathcal{I}$$

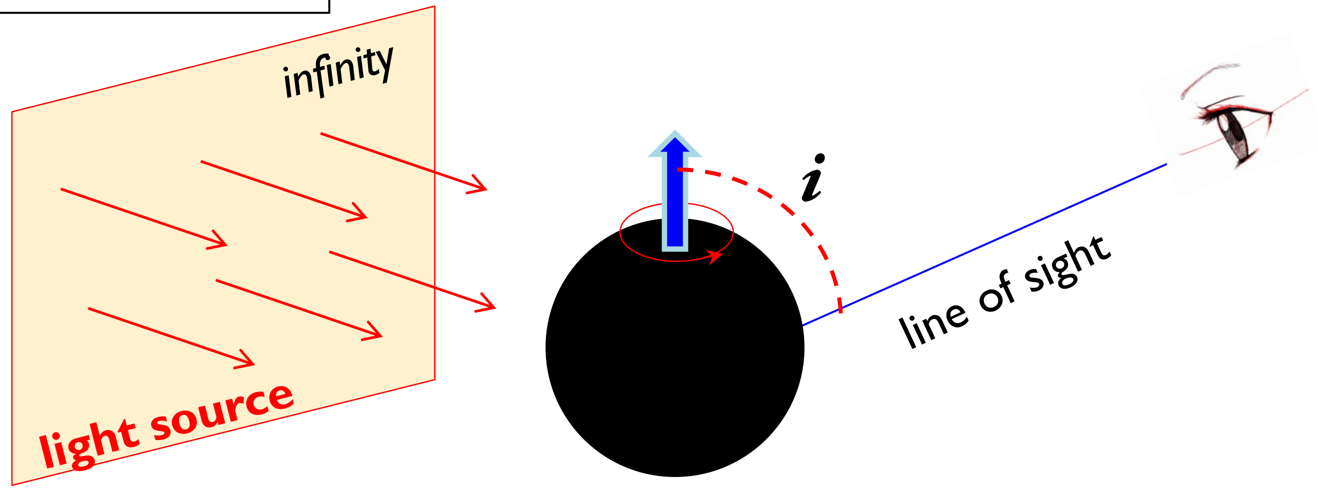
$$\Theta := \mathcal{I} - (a\sin\theta - \xi\csc\theta)^2$$

integration constants

$$\xi = \frac{L_z}{E} \qquad \eta = \frac{\mathcal{Q}}{E^2}$$

$$\mathcal{I}(\xi, \eta) := \eta + (a - \xi)^2$$

Black Hole Shadow



photon orbit ⟺ (ξ, η)

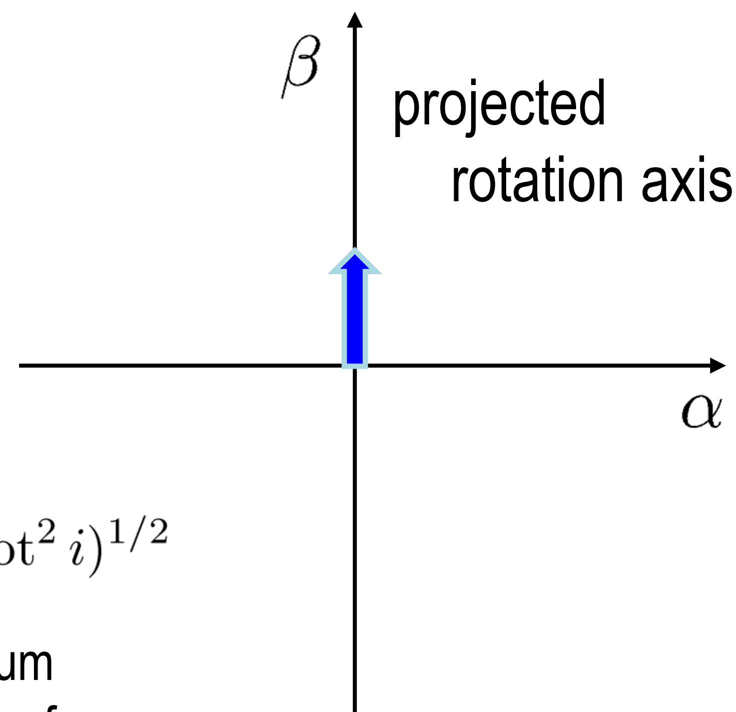
⇕ asymptotic behaviour

celestial coordinates (α, β)

$$\alpha = \lim_{r\to\infty} \frac{-rp^{(\phi)}}{p^{(t)}} = -\xi \csc i$$

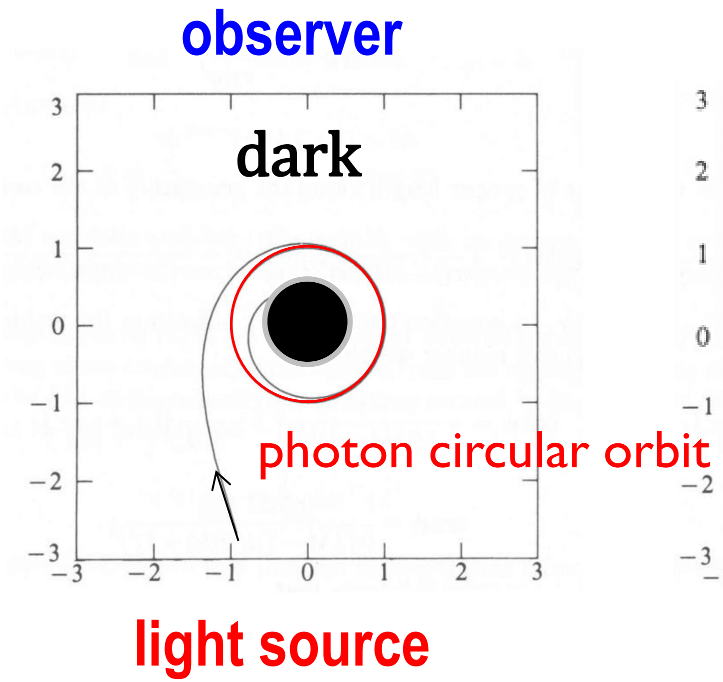
$$\beta = \lim_{r\to\infty} \frac{rp^{(\theta)}}{p^{(t)}} = (\eta + a^2 \cos^2 i - \xi^2 \cot^2 i)^{1/2}$$

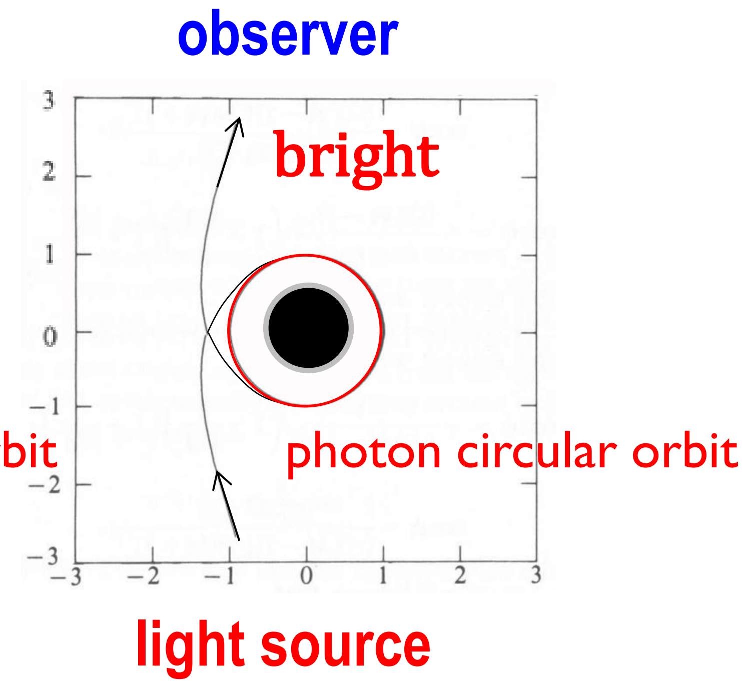
$p^{(a)}$: **tetrad components** of 4-momentum
w.r.t. locally non-rotating reference frame



How to find a black hole shadow

Schwarzschild spacetime





The boundary of shadow

For Kerr spacetime

circular orbit on the equatorial plane

$$r_{\rm circ}^{(\pm)} = 2M\left\{1+\cos\left[\frac{2}{3}\cos^{-1}\left(\mp\frac{a}{M}\right)\right]\right\}$$

$$\xi_{\rm circ}^{(\pm)} = \frac{\left[(r_{\rm circ}^{(\pm)})^2+a^2\right]}{a} - \frac{2r_{\rm circ}^{(\pm)}\Delta(r_{\rm circ}^{(\pm)})}{a(r_{\rm circ}^{(\pm)}-M)}$$

$$\eta = 0$$

+: direct orbit

–: retrograde orbit

$|\xi| < |\xi_{\rm circ}^{(\pm)}|$ ⇨ dark ⇨ shadow

For naked singularity

$$\tilde{r}_{\rm circ}^{(-)} = 2M\left\{1+\cosh\left[\frac{2}{3}\cosh^{-1}\left(\frac{a}{M}\right)\right]\right\}$$

–: retrograde orbit

$\eta \neq 0$

circular orbit ⇨ spherical photon orbit

3-dimensional but r = constant

circular orbit on the equatorial plane → spherical orbit $r = r_{\rm sph}$

$\mathcal{Q} = 0$ $\quad$ $\mathcal{Q} \neq 0$

$\eta = 0$ $\quad$ $\eta \neq 0$

one parameter

$$\xi_{\rm sph} = \frac{\left[(r_{\rm sph})^2 + a^2\right]}{a} - \frac{2 r_{\rm sph} \Delta(r_{\rm sph})}{a(r_{\rm sph} - M)}$$

$$\eta_{\rm sph} = -\frac{r_{\rm sph}^3 \left[r_{\rm sph}(r_{\rm sph} - 3M)^2 - 4a^2 M\right]}{a^2 (r_{\rm psh} - M)^2}$$

boundary values of shadow

$$\mathcal{I} = \frac{4 r_{\rm sph}^2 \Delta(r_{\rm sph})}{(r_{\rm sph} - M)^2}$$

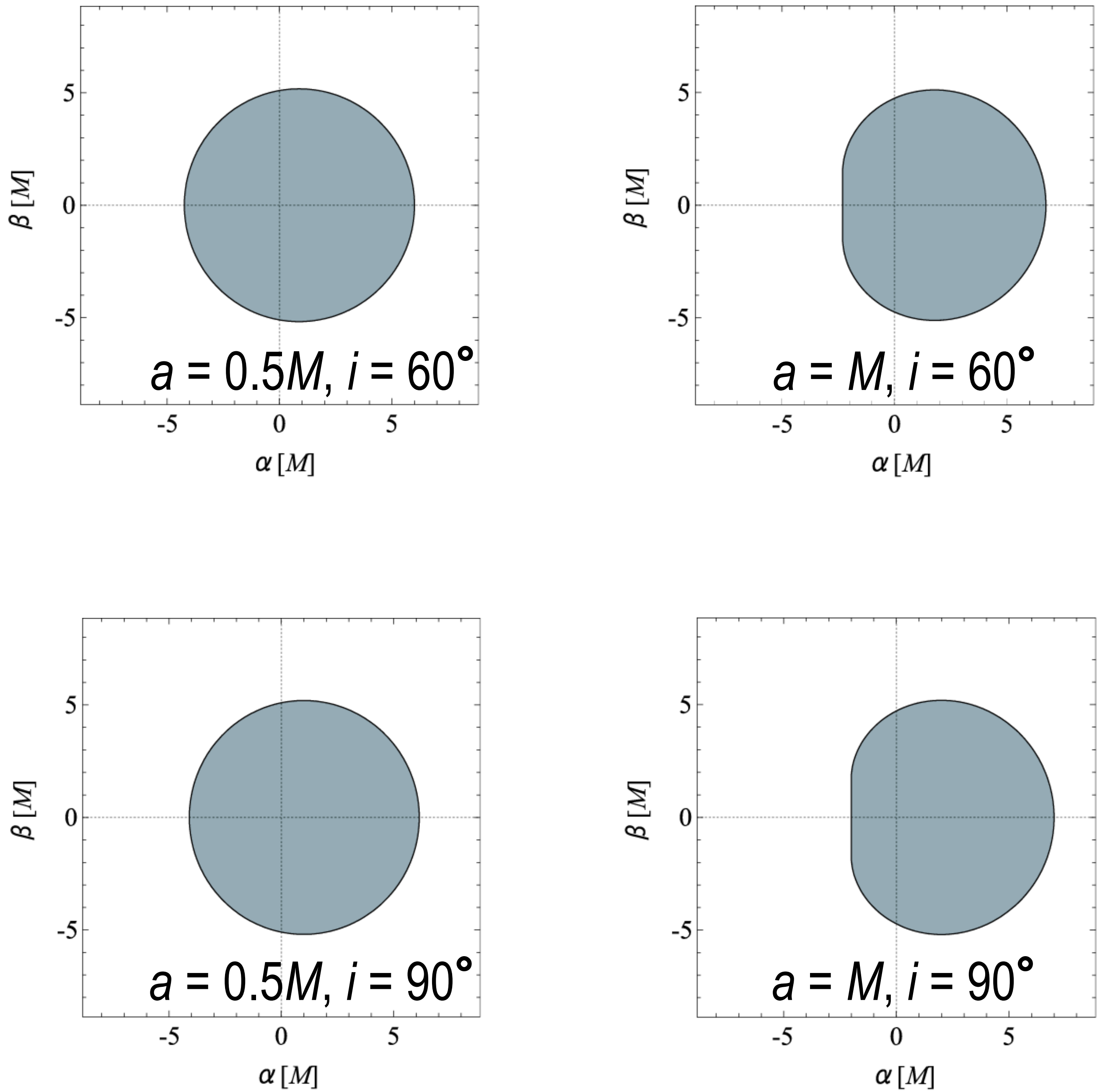
⇩

$$\alpha_{\rm sph} = -\xi_{\rm sph} \csc i$$

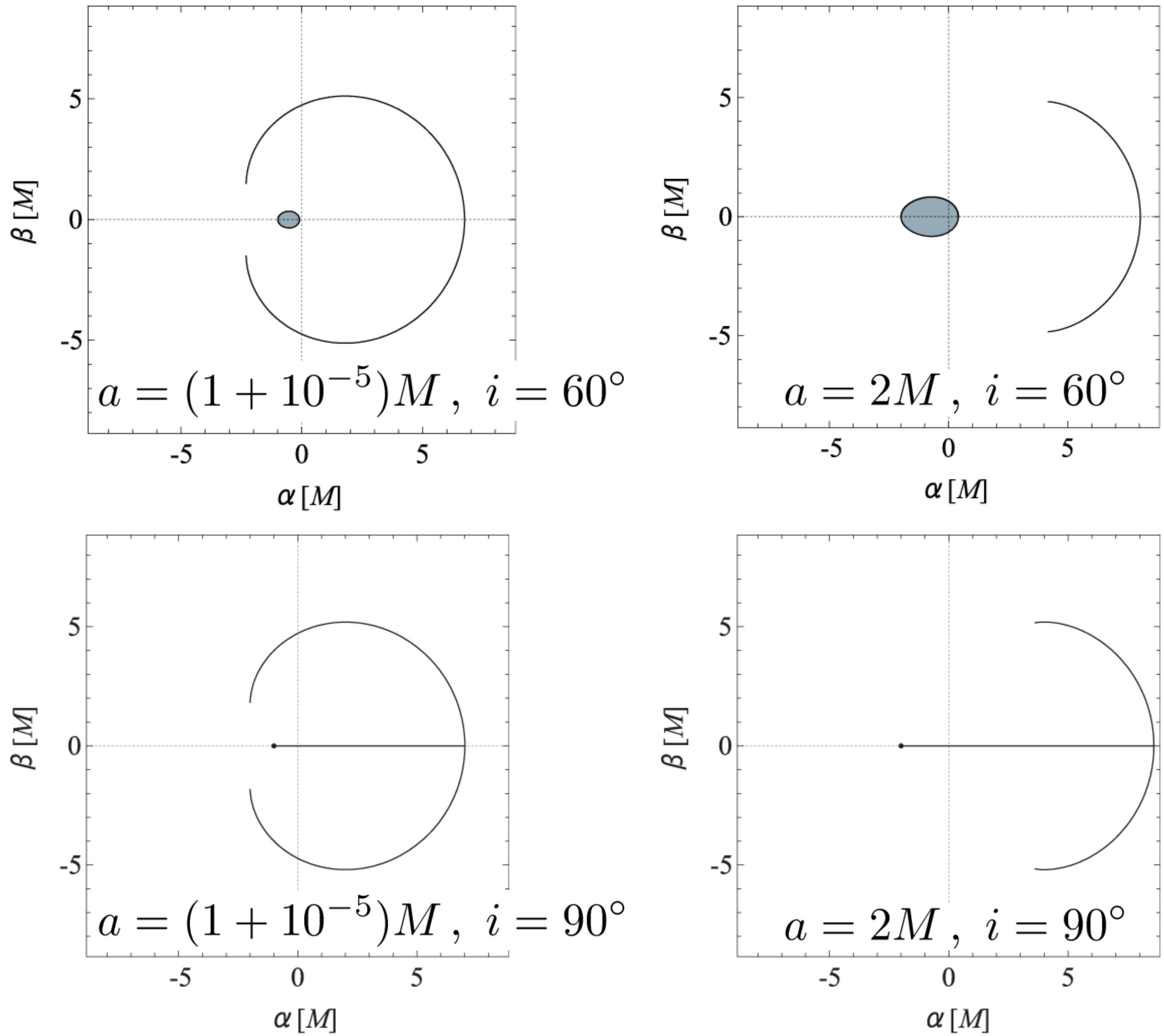
$$\beta_{\rm sph} = (\eta_{\rm sph} + a^2 \cos^2 i - \xi_{\rm sph}^2 \cot^2 i)^{1/2}$$

boundary of shadow

Kerr BH



Kerr naked singularity



How to determine a spin of a black hole

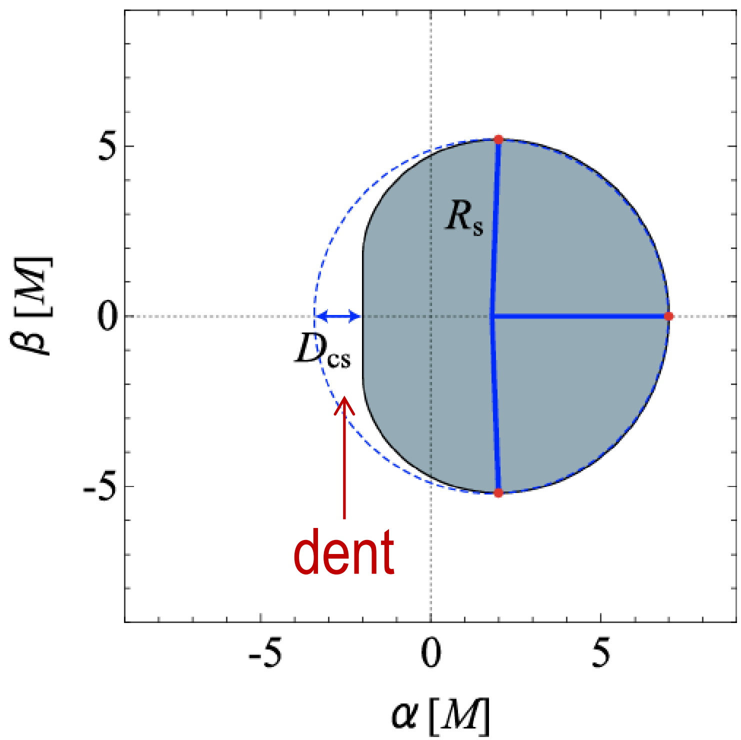
spin a , mass M + inclination i ⇒ Observed shape of a shadow

(a , i) ⇐ ? assuming M is fixed

Parameterization of shadow

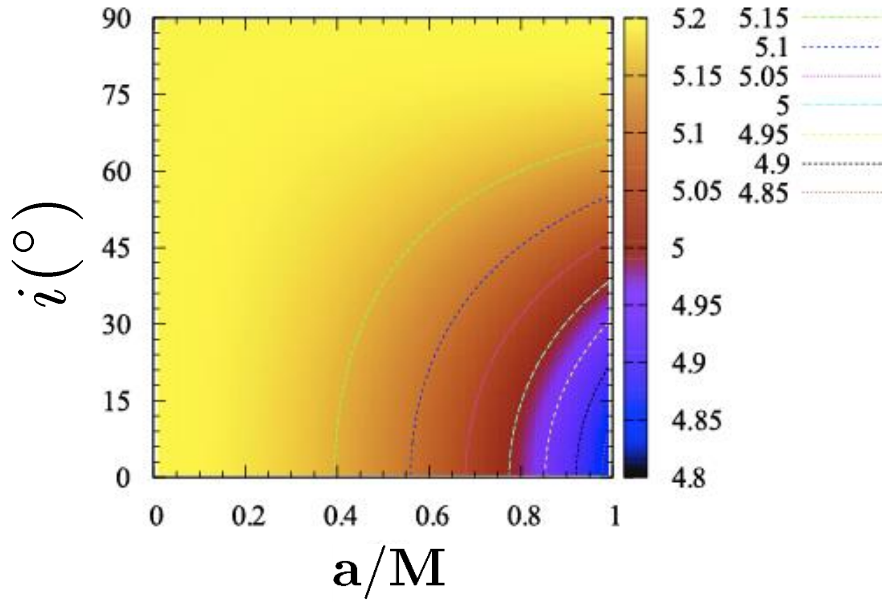
radius R_s

distortion $\delta_s := \dfrac{D_{cs}}{R_s}$

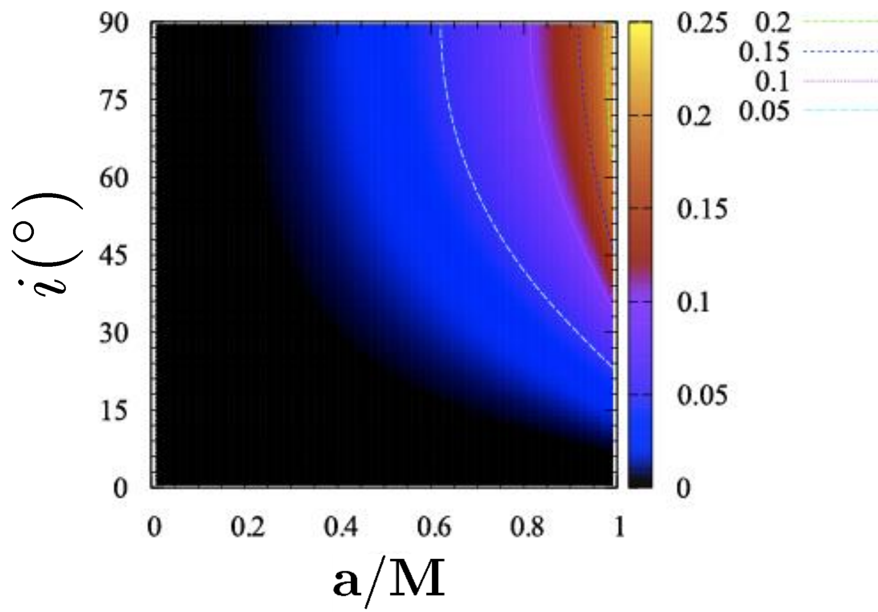


Contour maps of two parameters

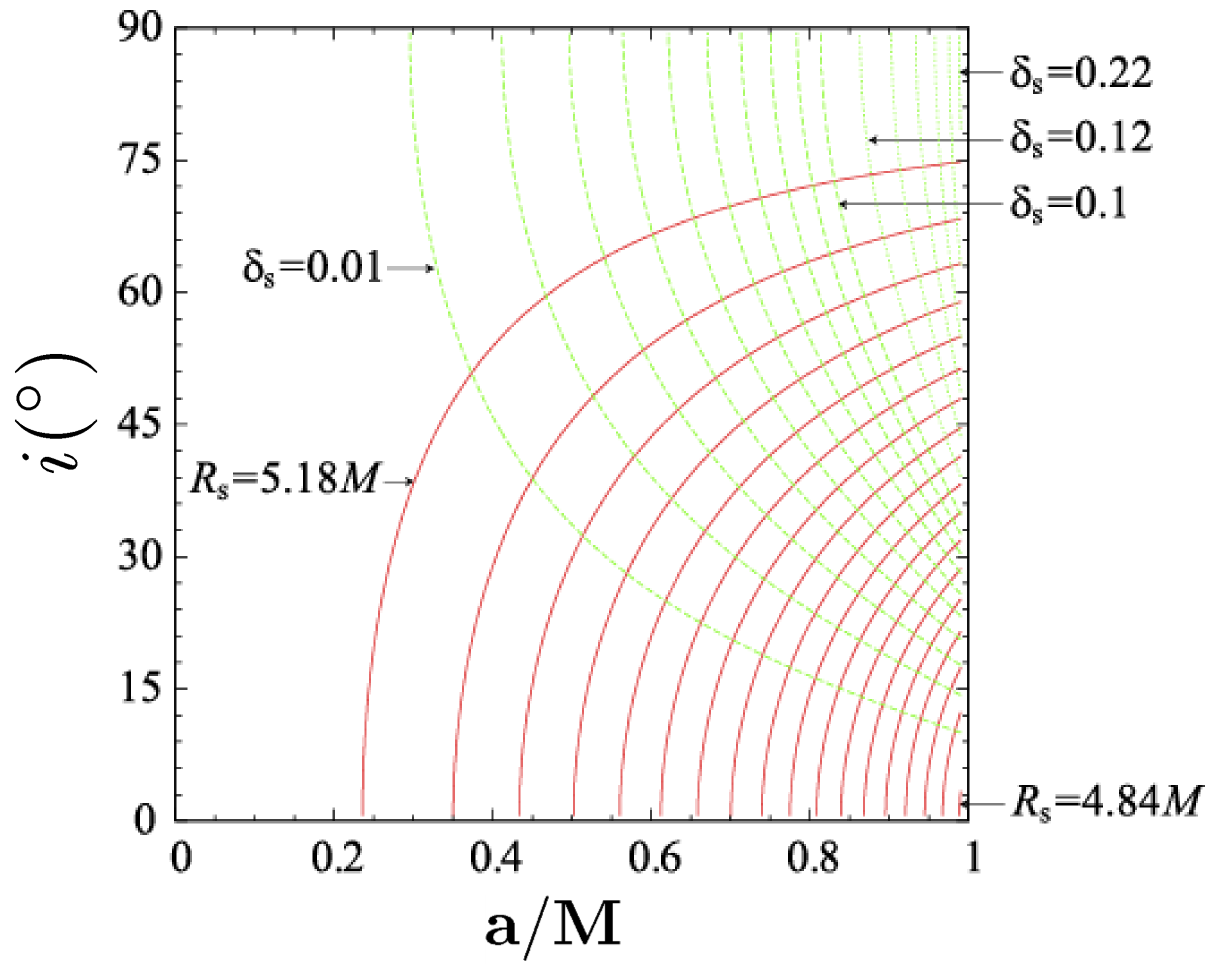
radius



distortion



Two contour maps



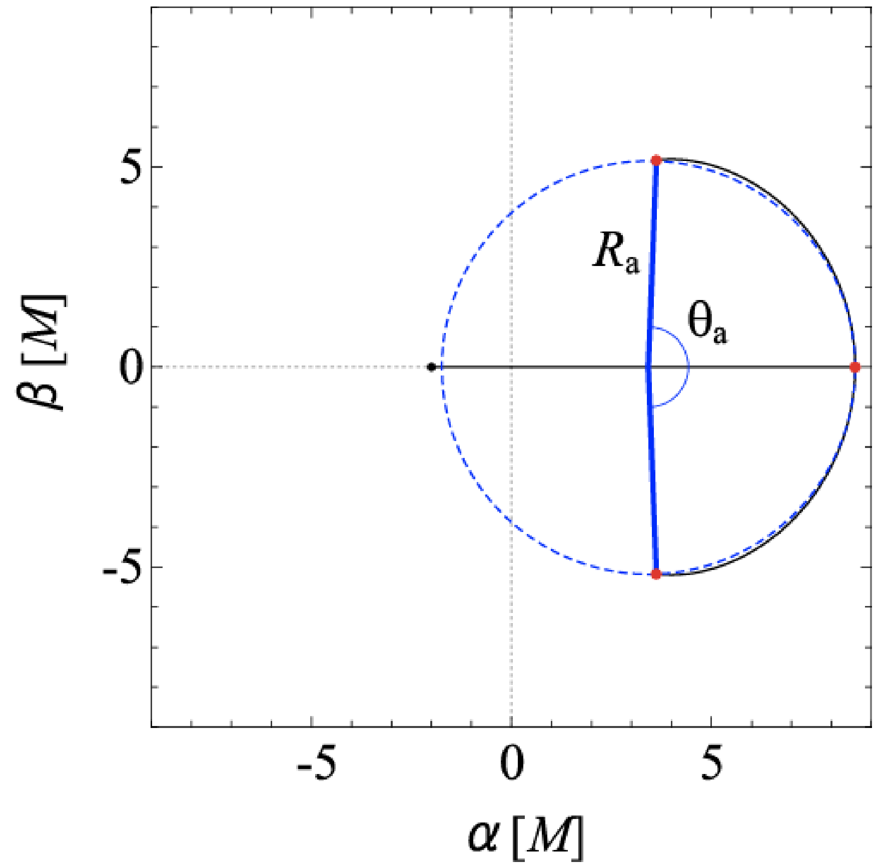
From observation of radius and distortion
we can evaluate the spin of BH *a* and the inclination angle *i* .

Kerr naked singularity

Parameterization of the arc

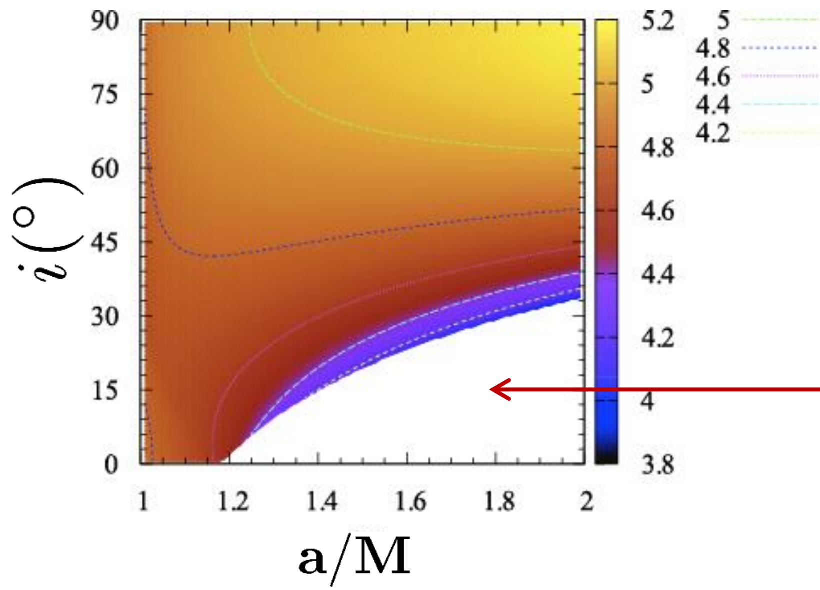
radius R_a

angle θ_a



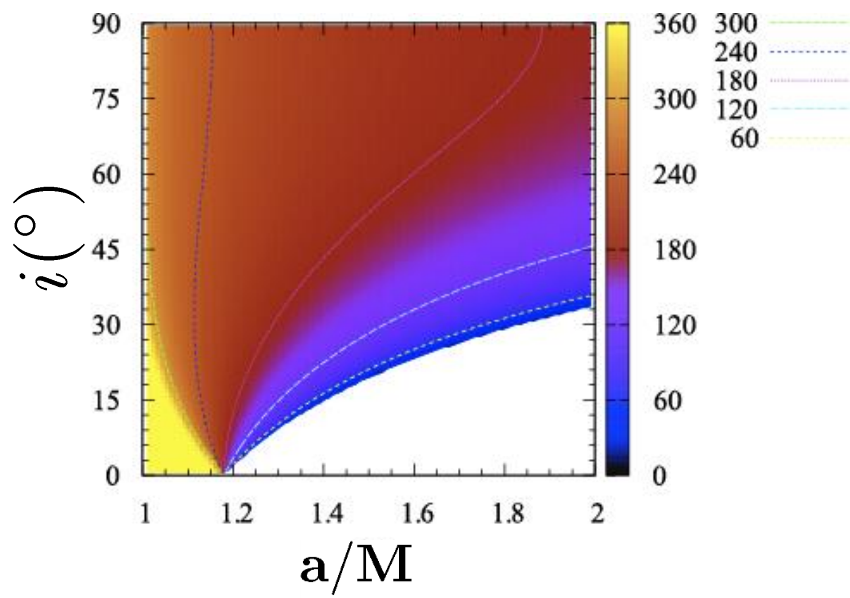
Contour maps of two parameters

radius

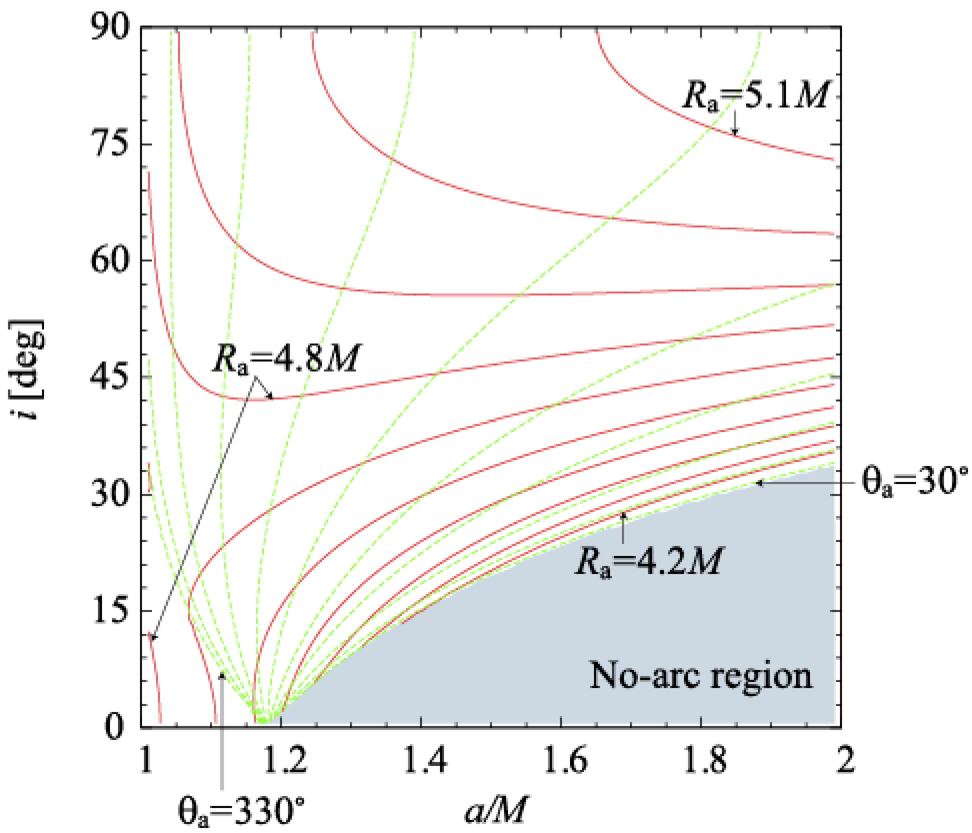


no arc

angle



Two contour maps

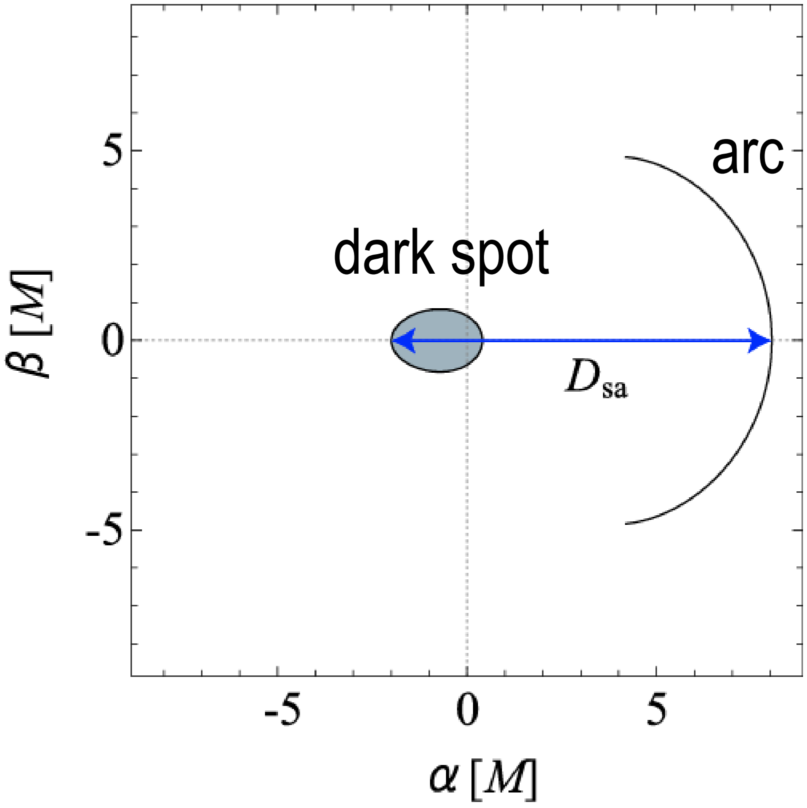


The contours are degenerated. It makes difficult to determine *a* and *i*

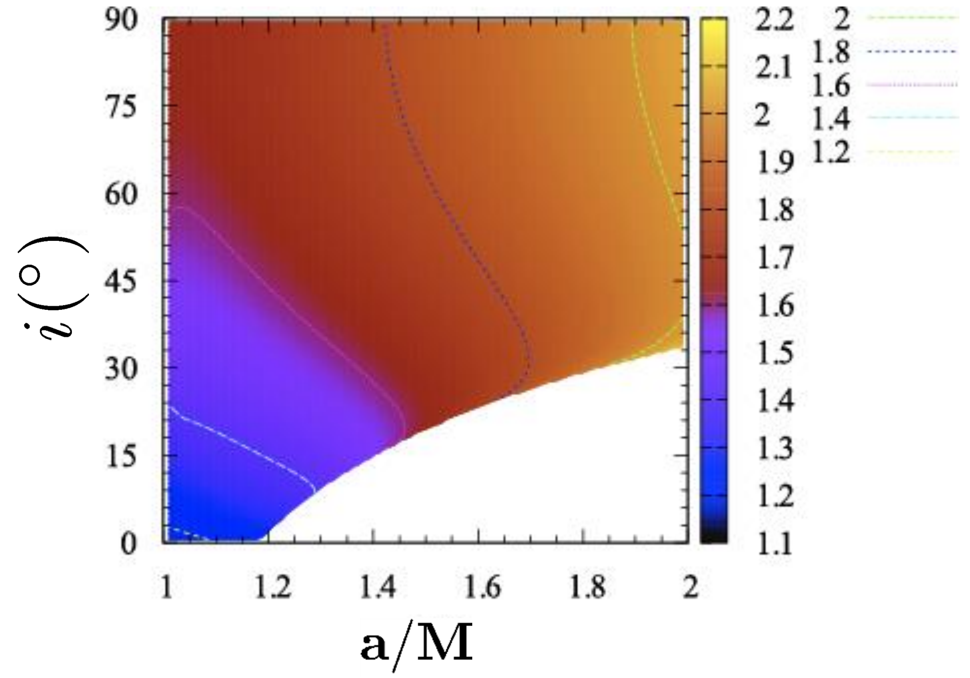
Different parameterization

radius R_a

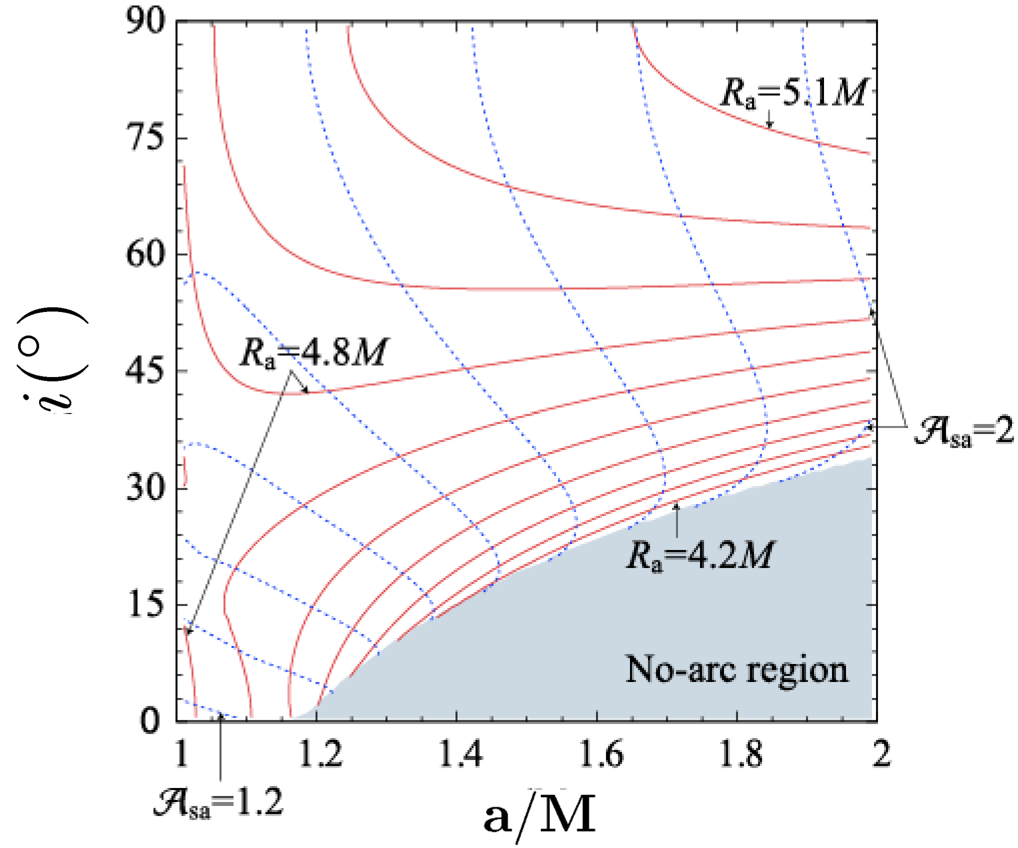
separation distance D_{sa}



Contour map of separation distance



Two contour maps of radius and separation distance



From observation of radius and separation distance
we can evaluate the spin *a* and the inclination angle *i* .

- **From observation of the shadow of a black hole or a naked singularity, we can evaluate the spin *a* and the inclination angle *i* .**

Flux density ⟹ **Brightness around a shadow**

More information about a black hole

More realistic situation

Surrounding matter   Accretion disc

Magnetic field

Dynamical system

Collision of black holes

Formation of a naked singularity

Collision of multi black holes ?

Kastor-Traschen (1993)

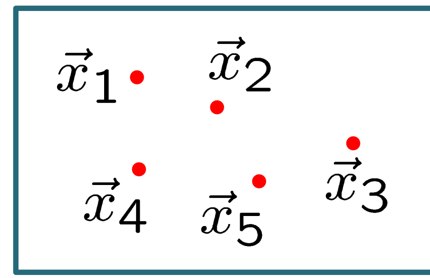
Multi-Extreme RNdS BHs

BHs in de Sitter spacetime

$$ds^2 = -V^{-2}dt^2 + U^2 a^2 d\vec{x}^2$$

$$a = e^{Ht} \qquad H^2 = \frac{\Lambda}{3}$$

$$U = V = 1 + \frac{1}{a}\sum_i \frac{M_i}{r_i}$$

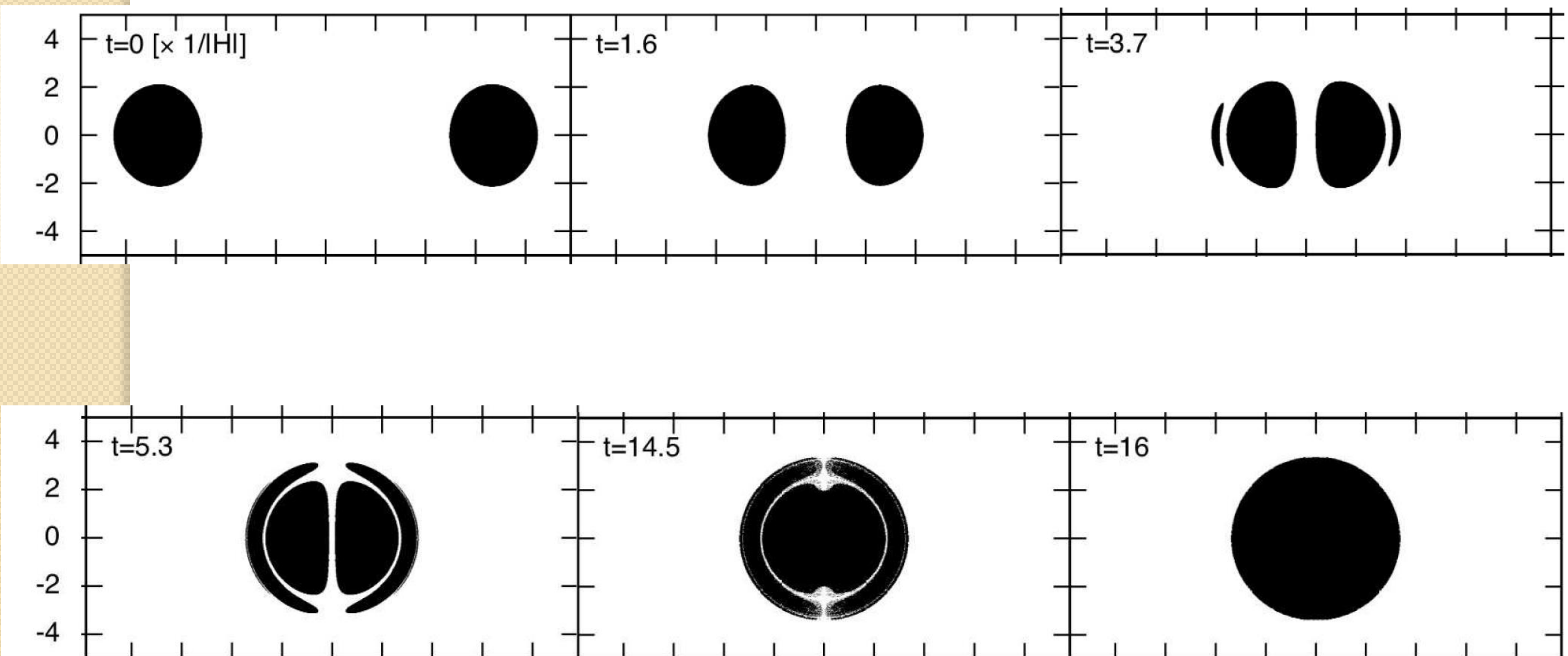


$$r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}$$

Time-dependent Contracting universe (time reversal)

Shadow of BH collision (KT solution)

Nitta, Chiba, Sugiyama ('11)



Testing GR

- Kerr black hole

  black hole uniqueness theorem in GR

- non-Kerr black hole

  Modified gravity

  Exotic matter

  ⇨ **Distinguishable ?**

Summary

- From observation of the shadow of a black hole or a naked singularity, we can evaluate the spin *a* and the inclination angle *i* .

- Studying more realistic situation and testing GR are interesting